OPTIMAL HYDRAULIC STRUCTURES PROFILES FOR IMPRECISE SEEPAGE HEAD AND PROBABILISTIC EXIT GRADIENT UNDER CLIMATIC UNCERTAINTY

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ABSTRACT

Hydraulic structures especially diversion works are founded on permeable foundation across rivers to facilitate flow into irrigation canals. There is, however, no procedure to fix the basic parameters such as depth of sheet piles/cutoffs and the length and thickness of floor, in a cost-effective manner. Changes in hydrological and climatic factors may alter the design seepage head of the hydraulic structures and also alter hydrogeologic properties of foundation formations. The variation in seepage head affects the downstream sheet pile depth, overall length of impervious floor, and thickness of impervious floor. The exit gradient, which is considered the most appropriate criterion to ensure safety against piping on permeable foundations, exhibits non linear variation in floor length with variation in depth of down stream sheet pile. These facts complicate the problem and increase the non linearity of the problem. However, an optimization problem may be formulated to obtain the optimum structural dimensions that minimize the cost as well as satisfy the exit gradient criteria. The optimization problem for determining an optimal section for the weirs or barrages normally consists of minimizing the construction cost, earth work, cost of sheet piling, length of impervious floor etc. The subsurface seepage flow is embedded as constraint in the optimization formulation.

Statistical techniques have been traditionally used to deal with parametric variation in model inputs, but these require substantial hydrogeologic explorations data for estimates of probability distributions. In the presence of limited, inaccurate or imprecise information, simulation with fuzzy numbers represents an alternative tool to handle parametric uncertainty. Fuzzy sets offer an alternate and simple way to address uncertainties even for limited exploration data sets. In the present work, the optimal design is obtained assuming a deterministic value of seepage head, in optimization model. Uncertainty in seepage head is then characterized using fuzzy numbers. The fuzzified nonlinear optimization formulation (NLOF) is then solved using optimization algorithm on MATLAB platform. Alternatively, the optimization formulation is also solved considering probabilistic value of safe exit gradient. Uncertainty in design, and hence cost from uncertain seepage head are quantified using fuzzy numbers.

Keywords: barrage design; nonlinear optimization; fuzzy numbers; seepage head; exit gradient, probabilistic exit gradient, climatic uncertainty

INTRODUCTION

Hydraulic structures such as diversion head works are costly water resources projects. Barrages are integral parts of major diversion head works. A cost effective design of barrages results into cost effective design of overall diversion head works. A safe and optimal design of these hydraulic structures is always being a challenge to water resource researchers. The hydraulic structure on alluvial soils is subjected to subsurface seepage. The seepage head causing the seepage vary with variation in flows. Design of hydraulic structures should also insure safety against seepage induced failure of the hydraulic structures.

Design and construction of diversion head works has always been a matter of concern for hydraulic researchers and field engineers. A typical layout of diversion head works is shown in Fig. 1. The variation in seepage head affects the downstream sheet pile depth, overall length of impervious floor, and thickness of impervious floor. The exit gradient is considered the most appropriate criterion to ensure safety against seepage induced piping on permeable foundations [1][2]. It exhibits non linear variation in floor length with variation in depth of down stream sheet pile. These facts complicate the problem and increase the non linearity of the problem. However, an optimization problem may be formulated to obtain the optimum structural dimensions that minimize the cost as well as satisfy the safe exit gradient criteria.

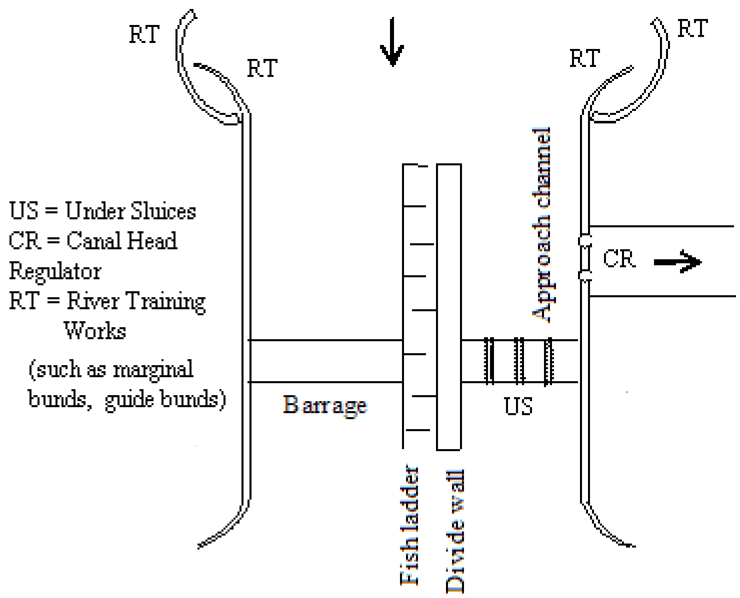

Fig.1. Layout of a diversion head works

The optimization problem for determining an optimal section for the weirs or barrages consists of minimizing the construction cost, earth work, cost of sheet piling, and length of impervious floor [3][4]. . Earlier work of discussed the optimal design of barrage profile for single deterministic value of seepage head [3]. This study first solve the of nonlinear optimization formulation problem (NLOP) which gives optimal dimensions of the barrage profile that minimizes unit cost of concrete work, and earthwork and searches the barrage dimension satisfying the exit gradient criteria. The work is then extended to characterize uncertainty in design due to imprecise seepage head which may resulted due to climatic changes and/or measurement errors. Uncertainty in design, and hence cost from uncertain head value are quantified using fuzzy numbers.

Subsurface Flow

The general seepage equation under a barrage profile may be written as:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \qquad (1)$$

This is well known Laplace equation for seepage of water through porous media. This equation implicitly assumes that (i) the soil is homogeneous and isotropic; (ii) the voids are completely filled with water; (iii) no consolidation or expansion of soil takes place; and (iv) flow is steady and obeys Darcy’s law.

For 2-dimensional flow, the seepage equation (1) may be written as:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0 \qquad (2)$$

The need to provide adequate resistance to seepage flow represented by equation (1) both under and around a hydraulic structure may be an important determinant of its geometry [5].The boundary between hydraulic structural surface and foundation soil represents a potential plane of failure.

Stability under a given hydraulic head could in theory be achieved by large combination of vertical and horizontal contact surfaces below the structure provided that the total length of the resultant seepage path were adequately long for that head [5][6]. In practical terms, the designer must decide on an appropriate balance between the length of the horizontal and vertical elements. Present work utilized Khosla's Method of independent variables [2] to simulate the subsurface behavior in the optimization formulation. Method of independent variables is based on Schwarz-Christoffel transformation to solve the Laplace equation (1) which represents seepage through the subsurface media under a hydraulic structure. A composite structure is split up into a number of simple standard forms each of which has a known solution. The uplift pressures at key points corresponding to each elementary form are calculated on the assumption that each form exists independently. Finally, corrections are to be applied for thickness of floor, and interference effects of piles on each others.

Exit Gradient

An explicit check is for the stability of the hydraulic structure for soil at the exit is devised by Khosla in the from of exit gradient [1].

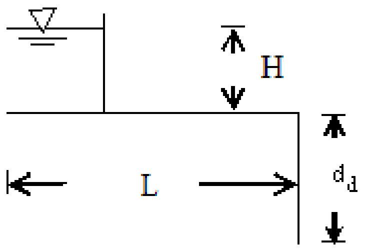

Fig.2. Definition sketch for parameters used in exit gradient

The exit gradient at downstream point for the simple profile as shown in Fig. 2 is given by Khosla as follows:

$$G_E = \frac{H}{d_d}\frac{1}{\pi\sqrt{\lambda}} \qquad (3)$$

where $\lambda = \frac{1}{2}[1+\sqrt{1+\alpha^2}]$; $\alpha = \frac{L}{d_d}$; L is total length of the floor; and H is the seepage head.

Equation (3) gives G_E equal to infinity for no sheet pile at the downstream side of the floor. Therefore, it is necessary that a vertical cutoff (sheet pile) be provided at the downstream end of the floor. To prevent piping, the exit gradient is kept well below the critical values which depend upon the type of soil.

The uplift pressures and exit gradients are two basic essentials which must be considered while designing a hydraulic structure. The uplift pressures must be balanced by a suitable thickness of floor at different points, and the exit gradients by a suitable depth of pile line at the downstream end of the floor. Any optimal design of hydraulic structure must incorporate these two in the optimization formulation.

OPTIMAL DESIGN METHODOLOGY

Optimization Model

Minimize $\quad C(L, d_1, d_d) = c_1(f_1) + c_2(f_2) + c_3(f_3) + c_4(f_4) - c_5(f_5) \qquad (4)$

Subject to $$\Pr ob\left[\frac{H}{d_d \pi \sqrt{\lambda}} \le C\right] \ge \beta \tag{5}$$

$$L^{l} \le L \le L^{u} \tag{6}$$

$$d_1^{\ l} \le d_1 \le d_1^{\ u} \tag{7}$$

$$d_d^{\ l} \le d_d \le d_d^{\ u} \tag{8}$$

$$L, d_1, d_d \ge 0 \tag{9}$$

where C (L, d_1, d_d) is objective function represents total cost of barrage per unit width (Rs/m), and is function of floor length (L), upstream sheet pile depth (d_1) and downstream sheet pile depth (d_d); f_1 is total volume of concrete in the floor per unit width for a given barrage profile and c_1 is cost of concrete floor (Rs/m^3); f_2 is the depth of upstream sheet pile below the concrete floor and c_2 is the cost of upstream sheet pile including driving (Rs/m^2); f_3 is the depth of downstream sheet pile below the concrete floor and c_3 is the cost of downstream sheet pile including driving (Rs/m^2); f_4 is the volume of soil excavated per unit width for laying concrete floor and c_4 is cost of excavation including dewatering (Rs/m^3); f_5 is the volume of soil required in filling per unit width and c_5 is cost of earth filling (Rs/m^3).

Equation (5) is probabilistic constraint in which 'C' represents safe exit gradient as random variable. In the context of present formulation, the safe exit gradient is assumed to be random in nature and follow normal distribution for a given soil formation on which the hydraulic structure is constructed with specified mean and standard deviation [7]. Symbol β (0 < β < 1) represents the vector of confidence levels of probability for satisfying the defined constraints.

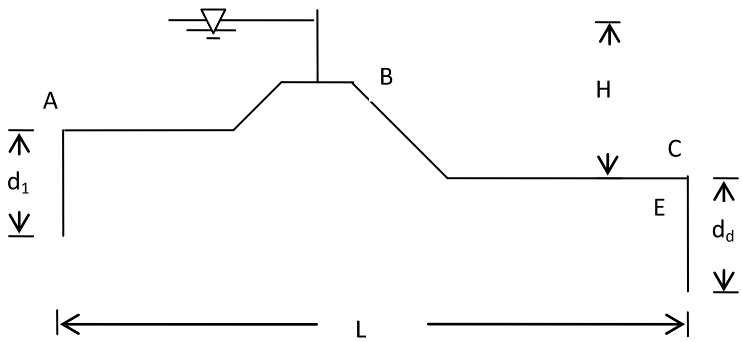

Fig. 3. Barrage profile utilized in performance evaluation of optimization method.

In the optimization formulation, for a give barrage profile and seepage head H (Fig.3), f_1 is computed by estimating thickness at different key locations of the floor using Khosla's merhod [1], and hence nonlinear function of length of floor (L), upstream sheet pile depth (d_1) and downstream sheet pile depth (d_d). Similarly f_4, and f_5 is nonlinear. The constraint represented by equation (10) is also nonlinear function of length of the floor and downstream sheet pile depth (d_2). Thus both objective function and constraint are nonlinear; make the problem in the category of nonlinear optimization program (NLOP) formulation, which are inherently complex.

Functional Parameters

For a given geometry of a barrage and seepage head H, the optimization model functional parameters f_1, f_2, f_3, f_4 and f_5 are characterized for the barrage profile shown in Fig.3. Intermediate sheet-piles are not effective in reducing the uplift pressures and only add to the cost of in reducing the uplift pressures and only add to the cost of the barrage [3]. In present work, no intermediate sheet piles are considered.

Probabilistic Constraint in Optimization Formulation

Equation (5) is a probability statement also called chance constrained which is not mathematically operational for algebraic solution. A deterministic or crisp equivalent of equation (5) needs to be derived.

Let, C is a random normal variable with mean and standard deviation E(C) and σ respectively.

Also, let L.H.S. (left hand side) of equation be represented by K. Let φ (y) be the distribution function of the random variable C. Then, since φ (y) is a monotonically non-decreasing function, also called CDF (cumulative distribution function), the value of the corresponding variable is determined as [8]:

$$\phi^{-1}(\eta) = \{Max\ y \vert \Pr ob(b \geq y) \leq \eta\ ,\ 0 \leq\ \eta \leq 1 \tag{10}$$

Now, the constraints set in (5) can be expressed as:

$$\Pr ob[K \leq C] = \Pr ob\left[\frac{C - E(C)}{\sigma} \geq \frac{K - E(C)}{\sigma}\right] \tag{11}$$

For prescribed level β, above equation may be written as:

$$\Pr ob\left[\frac{C - E(C)}{\sigma} \geq \frac{K - E(C)}{\sigma}\right] \geq \beta \tag{12}$$

The deterministic equivalent of the constraints in equation (5) with $\leq$ type can be obtained as [8][9]:

$$K \leq E(C) + \phi^{-1}(\beta) \times \sigma \tag{13}$$

Let R.H.S. (right hand side) of equation (13) is represented by C_D. Equation (5) may be further simplified using equation (13) and $\lambda = \frac{1}{2}[1+\sqrt{1+\alpha^2}]$; where $\alpha = \frac{L}{d_d}$; L is total length of the floor; H is the seepage head ; d_1 is the upstream sheet pile depth; d_2 is downstream sheet pile depth; L^l, d_1^l, and d_d^l is lower bound on L, d_1 and d_d respectively; L^u, $d_1^{\,u}$, $d_d^{\,u}$ are upper bound on L, d_1 and d_d respectively. The constraint equation (5) may be written as follows after substituting the value of λ and R.H.S. of equation (13) :

$$L - d_d \left(\{2(\frac{H}{d_2 \pi (C_D)})^2 - 1\}^2 - 1 \right)^{1/2} \geq 0 \tag{14}$$

Optimization Procedure

The NLOPPC optimization model represented by equations (4)-(14) and the functional parameters embedded in the optimization model are solved using non-linear constrained optimization function 'FMINCON' of MATLAB.

FMINCON finds a constrained minimum of a function of several variables.

FMINCON attempts to solve optimization problems of the form:

min F(X)

subject to: A*X <= B, Aeq*X = Beq (linear constraints) (15)

C(X) <= 0, Ceq (X) = 0 (nonlinear constraints) (16)

LB <= X <= UB (lower and upper bounds) (17)

In MATLAB the solution may be represented by

X = FMINCON (FUN, X0, A, B, Aeq, Beq, LB, UB, NONLCON) (18)

subjects the minimization to the constraints defined in NONLCON. The function NONLCON accepts X and returns the vectors C and Ceq, representing the nonlinear inequalities and equalities respectively. FMINCON minimizes FUN such that C(X)<=0 and Ceq(X)=0.

The basic steps employed in solution procedure may be presented as follows:

(i) Specification of parameters (decision variables) of problem domain in optimization formulation and confidence level of probabilistic variable.
(ii) For a specified seepage head, simulate seepage flow to find f_1, f_2, f_3, f_4, and f_5 parameters in optimization formulation in FUN.
(iii) Assign constraints in CONFUN.
(iv) Run FMINCON with assumed starting point (X0) and lower and upper bounds of decision variables.

Table 1. Barrage profile in conventional method utilized for performance evaluation as shown in Figure 1.

Physical parameters	Values (meters)
$^*L$	105.37
Y_{AC}	1.65
Y_{BC}	3.15
H	7.12
$^*d_1$	5.45
$^*d_2$	5.9

* Decision variables to be optimized

UNCERTAINTY CHARACTERIZATION IN THE OPTIMIZATION MODEL

Real-world problems, especially those that involve natural systems, such as soil and water, are complex and composed of many non-deterministic components having non-linear coupling. In dealing with such systems, one has to face a high degree of uncertainty and tolerate imprecision. There is a high degree of local soil variability, and imprecision in the determination of soil parameters and hydrological parameters like seepage head. Statistical techniques have been traditionally used to deal with parametric variation in model inputs, but these require substantial hydrogeologic explorations data for estimates of probability distributions. In the presence of limited, inaccurate or imprecise information, simulation with fuzzy numbers represents an alternative tool to handle parametric uncertainty. Fuzzy sets offer an alternate and simple way to address uncertainties even for limited exploration data sets.

Uncertainty in general comes in two forms: aleatory (stochastic, random natural variability or noncognitive) and epistemic (cognitive or subjective) [10]. Recently, Srinivasan et al. [11] identified these uncertainties in hydrogeological applications. Aleatory uncertainty refers to uncertainty that cannot be reduced by more exhaustive measurements or by a better model. Epistemic uncertainty, on the other hand, refers to uncertainty that can be reduced [12].

One of the milestones in the evolution of these new uncertainty theories is the seminal paper by Lofti A. Zadeh [13]. He proposed a new mathematical tool in his paper and called this new mathematical tool “fuzzy sets.” He proposed the concept of fuzzy algorithms in 1968 [14], and together with Bellman, proposed a new approach for decision-making in fuzzy environments in 1970 [15]. Fuzzy set theory has been recently applied in various fields for uncertainty quantification [16][17][18][19].

The transformation method presented by [17] uses a fuzzy alpha-cut (FAC) approach based on interval arithmetic. The uncertain response reconstructed from a set of deterministic responses, combining the extrema of each interval in every possible way unlike the FAC technique where only a particular level of membership (α -level) values for uncertain parameters are used for simulation.

Fuzzy modeling of uncertainty for hydrogeologic parameters such as exit gradient and seepage head is based on Zadeh’s extension principle [14] and transformation method (TM) [17]. In present study only seepage head is considered to be imprecise. Input seepage head as imprecise parameter, is represented by fuzzy numbers. The resulting output i.e. minimum cost obtained by the optimization model is also fuzzy numbers characterized by their membership functions. The reduced TM (Hanss, 2002) is used in the present study. The measure of uncertainty used is the ratio of the 0.1-level support to the value of which the membership function is equal to 1 [20].

RESULTS AND DISCUSSION

Earlier (mid 19th century), weirs and barrages have been designed and constructed in India on the basis of experience using the technology available at that period of time. Some of them were based on Bligh’s creep theory, which proved to be unsafe and uneconomical. Comparison of the parameters of these structures with the proposed approach is, thus, not justified. Therefore, a typical barrage profile, a spillway portion of a barrage, is chosen for illustrating the proposed approach as shown in Fig. 3. The barrage profile shown in Fig. 3 and parameters values given Table 1 is solved employing the methodology presented in this work. The barrage profile is first designed by the conventional method based on Khosla’s 2-D seepage analysis, in which the depth of sheet-piles is limited from scour considerations and the floor length is established to achieve a permissible exit gradient. The barrage profile is then optimized using the proposed approach with the same relative prices of materials used in the conventional method. In the optimization approach the depth of sheet-piles determined from scour considerations is taken as a lower bound (3.0 m), and the upper bound is set from practical considerations and limited to 12.0 m. Cost of material is taken from literature [3].

Random variable, C, is assumed to be normal distribution with mean value of 0.125 and standard variation 0.0015. The optimization results are obtained with confidence level (β values) at 95 percent. Results obtained with only mean values (50 % confidence level is also presented in Table 2. These results reveal that the NLOP optimization approach resulted in cost reduction compared to conventional design. The cost reduction is in the range of 38.28 to 39.25 percent as evident from Table 2. As the confidence level decreases, the cost increases due to lower value of safe exit gradient.

Table 2. Results of optimal design and conventional

Design Method	Barrage Profile			Cost (R/m)
	d_1 m	d_2 m	L m	$\times 10^4$
Conventional Method	5.45	5.9	105.37	13.4
NLOP				
β=0.95	12.00	11.81	40.00	8.14
β=0.50	12.00	12.99	41.008	8.27

Uncertainty Characterization

For characterization of uncertainty, seepage head is assumed to vary from 6.0m to 8.19m with central value of 7.12m i.e. almost 15 percent in triangular fuzzy numbers representation as shown in Fig. 4.

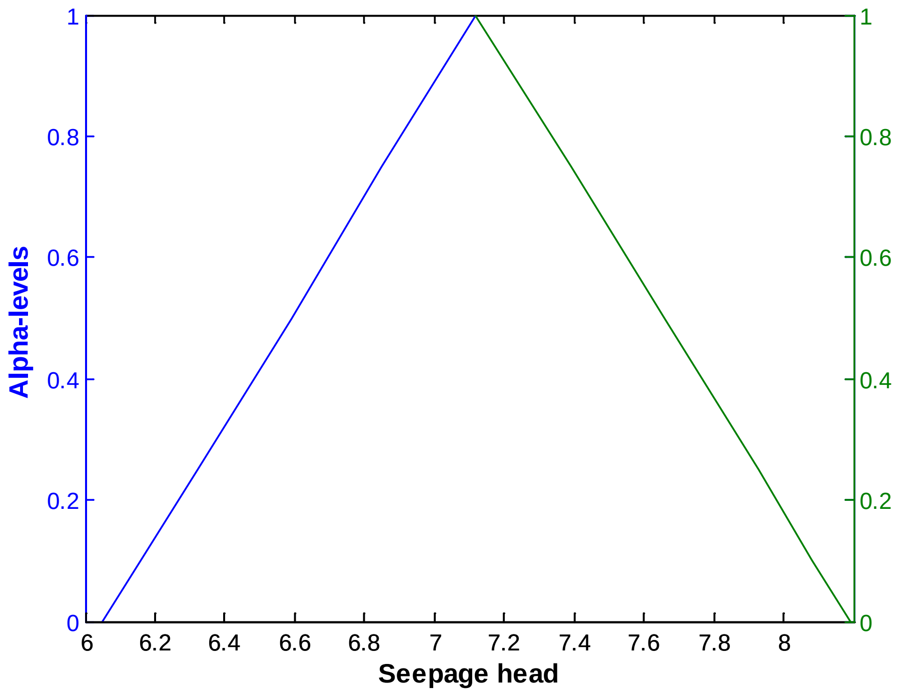

Fig.4. Representation of seepage head using triangular fuzzy numbers

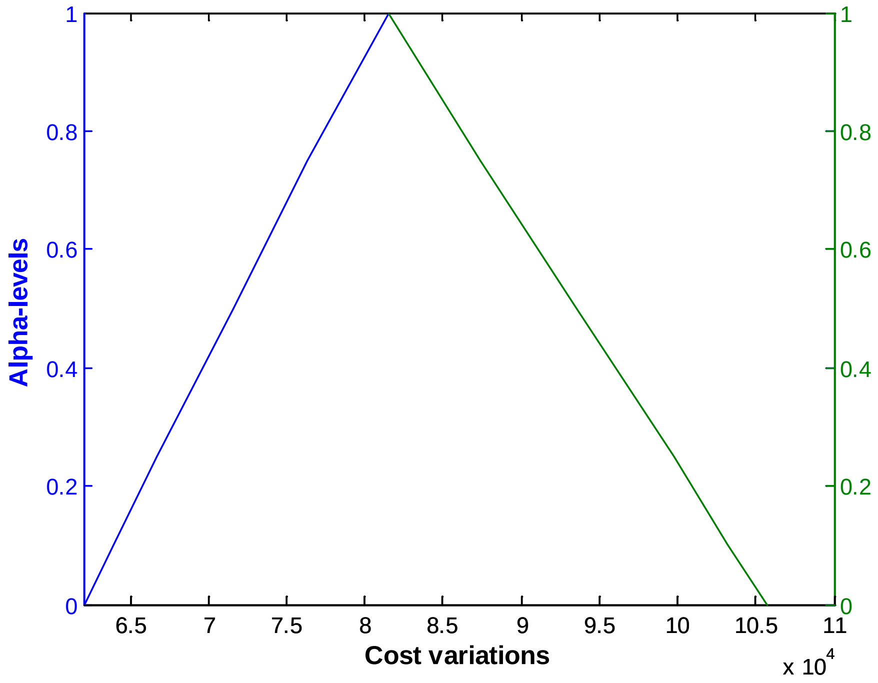

Fig.5. Cost variations of barrage with different α-levels

The result of variation in cost is corresponding different degree of membership for seepage head shown in Fig.5. The measure of uncertainty is found to be 48 percent. Since, left and right spread from central value of exit gradient is almost 15 percent, it can be concluded that uncertainty in seepage head reflects more uncertainty (more than 3 times) in cost.

CONCLUSIONS

The limited performance evaluation results show the potential applicability optimization based methodology to obtain the barrage profiles optimal costs at different confidence levels. At higher confidence level barrage profile results in lower cost than design based on mean values. However, it is required to evaluate the methodology on larger problems considering both surface and subsurface flows. The present work also demonstrates the fuzzy based framework for uncertainty characterization in optimal cost for imprecise hydrologic parameter such as seepage head which resulted due to measurement or climatic uncertainty. The uncertainty in cost is found not to be non linearly varying with uncertainty in seepage head. The optimization approach is equally valid for optimal design of other major hydraulic structures.

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